n = stage number
N = last stage
o = stream entering the system

LITERATURE CITED

1. Smoker, E. H., *Trans. Am. Inst. Chem. Engr.*, 34, 165 (1938).
2. Tiller, F. M., and R. S. Tour, *ibid.*, **40**, 317 (1944).
3. Tiller, F. M., *Chem. Eng. Progr.*, **45**, 391 (1949).
4. Martin, J. J., *A.I.Ch.E. J.*, **9**, 647 (1963).
5. Haldane, J. B. S., *Proc. Cambridge Phil. Soc.*, **28**, 234 (1932).
6. Milne-Thomson, L. M., "The Calculus of Finite Differences," p. 341, Macmillan, London, England (1933).
7. Knudsen, J. G., *Chem. Eng.*, **63**, 188 (April, 1956).
8. Lewis, W. K., *Ind. Eng. Chem.*, **14**, 492 (1922).

A Neglected Effect in Entrance Flow Analyses

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Flow in the entrance of a tube or channel has been the subject of numerous publications over the past one hundred years. The boundary value problem involved is the development of the well-known parabolic velocity distribution in a tube or between parallel plates, starting from a flat velocity profile. It has been suggested that this corresponds to what happens when a fluid having a very low turbulence intensity enters a tube or channel through a well-rounded converging duct, although as has been recently demonstrated by Wang and Longwell (*1*), the flat velocity profile boundary condition is definitely not a good model for the flow in which parallel plates are submerged in a fluid in uniform motion.

The hydrodynamic entry region was initially of interest in connection with viscosity measurement and has more recently been involved in attempts to establish a theory for the origin of turbulence in pipes and channels. In the meanwhile, it has provided an interesting diversion for a number of applied mathematicians. The author has been able to discover no less than twenty-five papers involving forty-three authors which deal with the theoretical analysis of laminar flow in the entrance to a channel or pipe with initially flat velocity profile. Except for the recent numerical solution of Wang and Longwell (*1*), all of these contributions have neglected a phenomenon which is of particular importance in applications of the results to stability analysis. Furthermore, this phenomenon, which may be referred to as the *persistence of parallel flows*, is of great importance in certain types of jet flows.

The model which has commonly been employed in the entry region problem for the flow close to the entrance is based on the simplified equation obtained by making boundary layer type assumptions in the equations of motion. Furthermore it is assumed that the velocity field may be represented as consisting of a core of uniform but accelerating fluid and a boundary layer at the wall. It is the author's opinion that such a model involving the hypothesis of an accelerating core cannot yield reliable information as to the details of the flow near the entrance. The argument is as follows.

Let us consider a flow of incompressible fluid in a tube or channel with the following boundary conditions:

$u = U$ at $x = 0$ (velocity initially uniform)

$u = 0$ at $r = R$ (no slip at the wall)

It may be argued from consideration of the contunity condition that a region of parallel, incompressible flow, such as we have at $x = 0$, can be altered only around its periphery. In other words, for a streamline to be deflected, fluid must flow with a velocity component normal to the direction of the parallel flow, and this can only occur where the flow is not parallel. This is the reason that there exists in a jet flow the so-called *potential core* in which the velocity is uniform. The mechanism by which the influence of the momentum gradient is transmitted into the core involves viscous shear. It must occur, then, at a finite rate depending on the viscosity of the fluid, its velocity, and the geometry of the confining walls. From an application of the Bernoulli equation to the flow in such a core, we conclude that the pressure gradient on the axis is zero up to a point at some finite value of x where the core of parallel flow is finally dissipated completely by shear at its periphery. This means that any pressure gradient measured at the wall of the entry region for small x is due entirely to radial pressure variations outside the core.

It can now be demonstrated that the existence of such a potential core in the entrance section precludes the applicability of the boundary layer model in this problem. First, the large pressure drops that have been measured at the wall in such flows (*2*) indicate the existence of a large radial pressure gradient. In addition, if the flow in a finite region in the neighborhood of the axis is just as it was at $x = 0$, and if the flow near the wall is being decelerated by viscous forces, continuity requires the existence of an intermediate region in which the velocity is higher than in the parallel flow core. The existence of such a maximum in the velocity would seem to have an extremely important effect on the stability of the flow, and stability analyses (*3, 4*), which have been based on boundary layer theory would thus seem to be of questionable validity.

Experimental measurements to date have not brought this effect to light, because they were taken either too far from the entrance or with insufficient accuracy. Wang and Longwell (*1*), however, have carefully solved, numerically, the complete set of equations for entrance flow in a channel, and their results (case 1) clearly show that for small x, the maximum in the velocity profile does not occur on the axis.

In conclusion, the author believes that if the velocity profile is flat at the entrance, there is a small but finite region near the entrance to a tube or channel in which the flow in a core of finite radius is just as it was at $x = 0$. This implies that the boundary layer type of analysis, based on the accelerating core model, is invalid for small values of x.

LITERATURE CITED

1. Wang, Y. L., and P. A. Longwell, *A.I.Ch.E. Journal*, **10**, 323 (1964).
2. Shapiro, A. H., R. Siegel, and S. J. Kline, *Proc. Natl. Congr. Appl. Mech.*, 773 (1954).
3. Hahneman, E., J. C. Freeman, and M. Finston, *J. Aerospace Sci.*, **15**, 493 (1948).
4. Tomita, T., *J. Phys. Soc. Japan*, **7**, 489 (1952).